UNIVERSITIES OF MANCHESTER LIVERPOOL
LEEDS SHEFFIELD AND BIRMINGHAM

Joint Matriculation Board

General Certificate of Education

MATHEMATICS. SYLLABUS I. GEOMETRY

ORDINARY

THURSDAY 18 NOVEMBER 1954, 9.30-12

Answer **all** *questions in* SECTION (A) *and any* **four** *questions from* SECTION (B).

In answer to Questions 1, 3 and 4 proofs are not required, but in calculations sufficient steps in the working must be shown to make clear how the calculation has been performed.

Mathematical tables provided.

Turn over

SECTION (A)

A1. (*a*) Calculate the size of an interior angle of a twelve-sided three-penny piece.

(*b*) Fig. 1 (not drawn to scale) represents two wheels of radii 8 in. and 3 in. Their centres A and B are 13 in. apart. A belt passes round both wheels in the manner shown so that the parts PQ, RS are straight. Calculate the length of belt not in contact with the wheels.

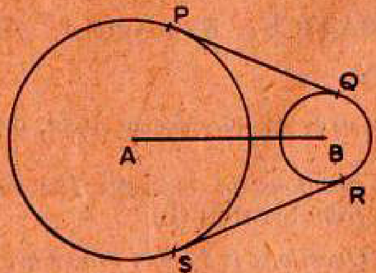

FIG. 1.

(*c*) Chords AB, CD of a circle intersect at an internal point X. If $AX=3$ in., $XB=4$ in., $CX=2$ in., calculate DX.

A2. (*a*) Prove that the diagonals of a parallelogram bisect one another.

(*b*) Two straight rods AX, BY rotate at equal rates in the clockwise direction about two fixed pivots A and B. If initially the rods are parallel but are pointing in opposite directions, prove that they will always be parallel.

A3. (*a*) In a triangle ABC, $AB=5$ in., $AC=6$ in., $BC=3$ in. *Calculate* the length of the projection of AB on AC.

(*b*) A regular hexagon is formed by removing triangles of equal area from the corners of an equilateral triangle of area 12 sq. in. Calculate the area of the hexagon.

A4. (*a*) The points A, B, C lie on the circumference of a circle with centre O so that A and C are on opposite sides of the radius OB. If $\angle AOB = 34°$ and $\angle OBC = 52°$, calculate (i) $\angle BAC$, (ii) $\angle ABC$.

(*b*) Two villages A and B are 4 miles apart. A factory is $3\frac{1}{2}$ miles from A and 2 miles from B. A hospital is to be built so that it is equidistant from A and B and $1\frac{1}{2}$ miles from the factory. *Using ruler and compasses only* and taking a scale of 1 in. to the mile, draw a plan showing the positions of the villages and the factory, and the two possible positions of the hospital.

In each position how far is the hospital from the villages?

Section (B)

Answer **four** *questions from* Section (B).

B5. Prove that if two chords of a circle are equidistant from the centre of the circle they are of equal length.

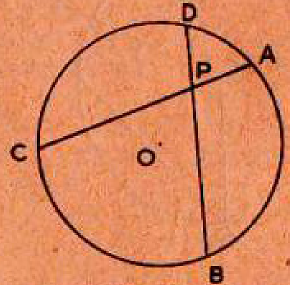

Fig. 2.

In Fig. 2 chords AC and BD of a circle with centre O intersect at P. If OP bisects $\angle BPC$, prove that (i) $AC = BD$, (ii) $AP = DP$.

B6. A straight line ABC is such that $AB = a$ units, $BC = b$ units, and $a > b$. Squares $ABPQ$ and $BCXY$ are drawn on the same side of AB. Prove that

(i) $CP^2 + QX^2 = 3(a^2 + b^2) = CQ^2 + PX^2$,

(ii) $\triangle AQC = \triangle AQB + \triangle BCY + \triangle PXY$,

(iii) Area of quadrilateral $QPXC = \frac{1}{2}CP^2$.

B7. Two chords of a circle intersect outside the circle. Prove that the rectangle contained by the parts of the one is equal to the rectangle contained by the parts of the other.

ABC is a triangle in which $\angle ABC = 90°$ and F is the midpoint of AB. The circle on AF as diameter cuts AC in G and the point of contact of a tangent from B to this circle is T. Prove that

(i) the quadrilateral $GFBC$ is cyclic,

(ii) $AG.AC = BT^2$.

B8. In Fig. 3, DY is a diameter of the circle with centre C, the lines DA and DB are equal and AD is perpendicular to DC.

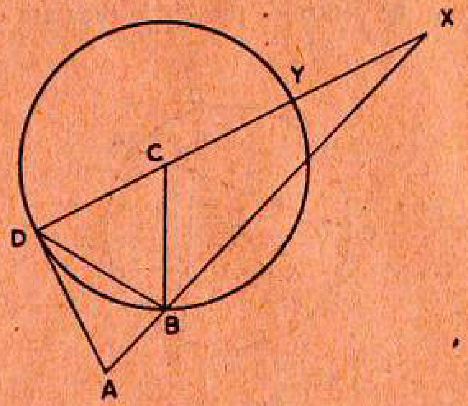

FIG. 3.

Prove that AD is a tangent to this circle and show that $\angle BCD = 4\angle DXA$. Prove also that the triangle BYX is isosceles.

B9. In Fig. 4 the circles with centres O and S touch at P and XK is a tangent at X. The straight line $MSLK$ is at right angles to KX. Prove that X, P and M lie on a straight line.

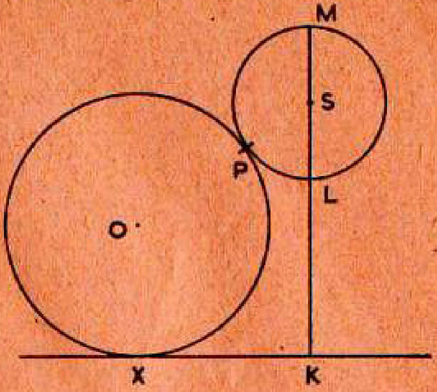

FIG. 4.

Construct a triangle MPK in which $MK=4$ in., $PK=2\frac{1}{2}$ in., and $MP=2$ in. *Using ruler and compasses only*, draw the circle which passes through the points P and M and whose centre, S, lies on the line MK. Construct also the centre O of the circle which touches this circle at P and also touches the line through K perpendicular to MK. Measure OP.

B10. Prove that if a straight line is drawn parallel to one side of a triangle, the other two sides are divided proportionally.

In a triangle ABC a line parallel to BC cuts AB, AC internally at Q, R respectively. A line through C parallel to BA cuts QR produced at O. If $AQ:AB=x:y$ and triangle ABC has area S, find the area of (i) triangle AQR, (ii) triangle COR, (iii) parallelogram $QOCB$.